Spontaneous Collapse of Supersymmetry

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Abstract

It is shown that, if generators of supersymmetry transformations (supercharges) can be defined in a spatially homogeneous physical state, then this state describes the vacuum. Thus, supersymmetry is broken in any thermal state and it is impossible to proceed from it by "symmetrization" to states on which an action of supercharges can be defined. So, unlike the familiar spontaneous breakdown of bosonic symmetries, there is a complete collapse of supersymmetry in thermal states. It is also shown that spatially homogeneous superthermal ensembles are never supersymmetric.

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1 Introduction

After more than a decade of discussions, a consensus has not yet emerged on the fate of supersymmetry in Minkowski space quantum field theory at finite temperatures. There exist contradictory statements in the literature, ranging from the assertion that supersymmetry is always broken spontaneously in thermal states [1], through arguments in favor of supersymmetry restoration at sufficiently high temperatures [2], up to the claim that supersymmetry can be unbroken at any temperature [3].

In this paper we reconsider the status of supersymmetry in a general setting including thermal states. After recalling some relevant facts from statistical mechanics whose

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significance is frequently ignored, we will establish the following result: If supercharges can be defined in a given spatially homogeneous state, then this state describes the vacuum. Hence supersymmetry is inevitably broken spontaneously in thermal states. As a matter of fact, this breakdown is much stronger than that of ordinary bosonic symmetries, where one can restore the symmetry by taking suitable averages of states with broken symmetry (an example being the spherical mean of a ferromagnetic state). In sharp contrast, such symmetrized thermal states do not exist in the case of supersymmetry, and we therefore refer to this fact as *spontaneous collapse* of supersymmetry (see below).

We also reconsider the notion of superthermal ensemble, described by a supertrace, and discuss its physical significance. It turns out that a spatially homogeneous supertrace cannot be supersymmetric unless it vanishes. Hence its behavior under supersymmetry transformations is known from the outset and does not provide any physically significant information.

In order to set the stage for our discussion, we first list some sources of confusion and indicate how these difficulties can be resolved. The general mathematical setting will be explained in Sec.2.

Necessity for thermodynamic limit

For a precise test of the spontaneous breakdown of a symmetry, it is necessary to study the thermodynamic (infinite volume) limit of the system under consideration. In the literature, the corresponding states are frequently treated as Gibbs ensembles and thermal averages of fields are presented in the form

$$\langle F \rangle = Z^{-1} \,\mathrm{Tr}\, \exp(-\beta H) F. \tag{1}$$

This is meaningful for systems confined in a finite volume (box). In the thermodynamic limit, however, this formula becomes meaningless, since $\exp(-\beta H)$ is then no longer a trace-class operator. Moreover, the thermodynamic limit may not be interchanged

with the spatial integrations involved in the definition of charge operators from current densities.

These mathematical facts are frequently ignored and have led to erroneous statements in the literature. This problem can be avoided, however, by characterizing the thermal averages as expectation functionals $\langle \cdot \rangle$ (called states in the following) on the field operators which satisfy the KMS (Kubo-Martin-Schwinger) condition [4]. This property survives in the thermodynamic limit and is a distinctive feature of thermal equilibrium states [5].

Necessity for renormalizing symmetry generators

Another problem, closely related to the above, is the following one: The definition of symmetry generators by volume integrals of conserved Noether currents, such as $H = \int d^3\boldsymbol{x}\ \theta_{00}(x)$ in the case of the generator of time translations, does not make sense in thermal states in the thermodynamic limit. In the given example, this is obvious if one considers the expectation value of H in a spatially homogeneous state with non-vanishing energy density: Infinite thermal systems contain an infinite amount of energy and an energy operator H can therefore not be defined in such states.

However, one can still define a generator $\hat{H}$ of time translations in thermal equilibrium states [4] by taking advantage of the fact that these states are mixed (not pure) and hence the basic fields do not form an irreducible set of operators in such states.[1] One can show that there exists a (state dependent) operator $\tilde{\theta}_{00}(x)$, commuting with all basic fields, such that the formal expression

$$\hat{H} = \int d^3\boldsymbol{x}\ \Big(\theta_{00}(x) - \tilde{\theta}_{00}(x)\Big) \tag{2}$$

can be given a precise meaning as an operator in the Hilbert space of the given thermal state. In commutators of $\hat{H}$ with the underlying fields the contribution of the tilde operator drops out, and hence $\hat{H}$ induces the same infinitesimal time translation as the

[1] In "thermo field dynamics" [6], one complements the basic fields by a set of auxiliary fields in order to deal with an irreducible set of operators. We do not make use of this formalism here.

ill-defined expression H. In a sense, the passage from H to $\hat{H}$ can be regarded as a renormalization to cancel out the infinities in H appearing in the thermodynamic limit.

The existence of operators commuting with the basic fields permits, in the case of unbroken symmetries, the construction of well-defined generators as described above, but it also introduces some element of arbitrariness: If one adds to $\hat{H}$ any operator commuting with the basic fields, one still obtains a generator of time translations. It is of interest here that, in the case of $\hat{H}$, one can remove this arbitrariness in a consistent manner by demanding that $\hat{H}$ annihilates the vector corresponding to the thermal state. (This shows, incidentally, that $\hat{H}$ does not have the meaning of energy since the energy content of a thermal state is fluctuating.) Thus the argument that supersymmetry must be spontaneously broken because thermal states carrying non-vanishing energy are not annihilated by H is in several respects inconclusive.

Spontaneous breakdown versus spontaneous collapse

The infinitesimal symmetry transformations of field operators arising from invariance properties of some Lagrangian induce linear mappings δ on the space of polynomials in these fields. In the following, we denote these polynomials generically by F. If δ corresponds to a symmetry of bosonic type, it satisfies the Leibniz rule

$$\delta(F_1 F_2) = \delta(F_1)\, F_2 + F_1\, \delta(F_2), \tag{3}$$

in an obvious notation. The analogous relation for symmetries of fermionic type is given in Sec.2.

The action of δ on the polynomials F is always meaningful and can be considered in any physical state. On the other hand, the question as to whether the symmetry is unbroken in the sense that δ can be represented in the form[2] $\delta(F) = [Q,\, F]$ depends on the physical situation under consideration. Referring to the concept of thermodynamic phases, we shall distinguish three significant cases: (i) pure phases with unbroken

[2]We refrain from introducing infinitesimal (Grassmannian) transformation parameters. In the case of fermionic symmetries, one then has to distinguish between bosonic operators and fermionic ones, replacing in the latter case the commutator by an anti-commutator, cf. Sec.2.

symmetry, (ii) pure phases with broken symmetry which can be restored, however, by proceeding to suitable mixed phases, and (iii) phases, pure or mixed, with spontaneously collapsed symmetry which cannot be restored.

Here the notion of pure and mixed thermodynamic phases should not be confused with that of pure and mixed states. A pure phase is characterized by sharp c-number values of macroscopic order parameters, which are statistically fluctuating in the case of mixed phases. Any thermodynamic phase, pure or mixed, is described by a mixed state; it is only in the case of vacuum states that the notions of pure phase and pure state coincide [5].

We recall that a state $\langle \cdot \rangle$ describing a pure thermodynamic phase has the cluster property (absence of long range order), i.e., it holds for any F_1, F_2 that

$$\langle F_1(\boldsymbol{x}) F_2 \rangle - \langle F_1(\boldsymbol{x}) \rangle \langle F_2 \rangle \to 0 \qquad \text{as } |\boldsymbol{x}| \to \infty. \tag{4}$$

Here $F(\boldsymbol{x})$ denotes the polynomial obtained from F by shifting the spacetime arguments of the underlying fields by $\boldsymbol{x}$. If a state $\langle \cdot \rangle$ describes a statistical mixture of phases, it can be decomposed into pure phases $\langle \cdot \rangle_\theta$,

$$\langle \cdot \rangle = \sum_\theta w_\theta \langle \cdot \rangle_\theta, \tag{5}$$

where θ is an order parameter labeling the pure phases and the weight factors w_θ are non-negative numbers which add up to 1. (In the case of a continuum of phases, the summation need be replaced by an integration with respect to a probability measure.) An important fact about this central decomposition of states is its uniqueness, which will be used later. For a thorough exposition of these facts, we refer to [5].

Returning now to the issue of symmetry, we consider any state $\langle \cdot \rangle$ describing a pure thermodynamic phase. By the reconstruction theorem [7], there exists a corresponding Hilbert space of vectors, describing this phase as well as all states which can be reached from it by the action of polynomials F in the fields. As indicated above, there are then the following three possibilities.

(i) There exists an operator Q on this Hilbert space which generates the symmetry transformation δ as described above. The symmetry is then said to be unbroken in this phase.

A frequently used test as to whether this situation is realized is given by:

$$\langle \delta(F) \rangle = 0 \quad \text{for all } F\,? \tag{6}$$

If the answer is affirmative, one can consistently define an operator Q with all desired properties. Yet it is sometimes overlooked that this test provides only a sufficient condition for the existence of such a Q. This can heuristically be understood if one thinks of a spatially inhomogeneous situation, e.g., a drop of liquid surrounded by gas. The corresponding state is then not invariant under translations and thus does not pass the test (6) for the infinitesimal translations δ. Nevertheless, translations can be defined on the underlying Hilbert space since the effect of shifting the drop can be described by the action of polynomials in the fields on the state vectors: One annihilates the constituents of the drop and creates them again at the shifted position.

In the case of spatially homogeneous states such as the vacuum, one can sometimes show that (6) provides also a necessary condition for the existence of Q. However, the arguments given to that effect in the literature (see, for example, [8]) are not conclusive in the case of thermal states because of the abovementioned difficulties in the definition of generators. The status of the test (6) thus needs a close re-examination.

(ii) The second possibility is that the symmetry is broken in the pure phase $\langle \, \cdot \, \rangle$, but one can proceed to a corresponding symmetrized mixed phase where the symmetry is restored and generators Q can be defined.

The familiar example of this kind already mentioned is the case of a ferromagnet. If $\langle \, \cdot \, \rangle_{\boldsymbol{\theta}}$ describes such a state with sharp direction $\boldsymbol{\theta}$ $(= (\vartheta, \varphi))$ of magnetization, spatial rotations cannot be defined on the corresponding Hilbert space (except for the rotations around the axis $\boldsymbol{\theta}$). But the mixed phase corresponding to the spherical average of these states, $\langle \, \cdot \, \rangle = \int d\boldsymbol{\theta} \, \langle \, \cdot \, \rangle_{\boldsymbol{\theta}}$, passes the test (6) with respect to infinitesimal rotations δ.

Hence, there exist generators for rotations on the corresponding "enlarged" Hilbert space. In other words, while the result of a rotation cannot be described in the Hilbert space of each pure phase since it cannot be accomplished by the action of polynomials in the local fields on the corresponding vectors, it is still meaningful to speak about the action of rotations on these states. Generators inducing this action can be defined in the Hilbert space of the symmetrized state $\langle \cdot \rangle$ since it comprises states with arbitrary directions of magnetization.

The situation described in this example illustrates the spontaneous breakdown of a symmetry: The symmetry is broken in a pure thermodynamic phase but restored in a suitable mixture where one can define corresponding generators. This situation prevails in the case of bosonic symmetries described by a (locally) compact group. It is this case which is usually taken for granted.

(iii) There is, however, a third possibility which is of relevance to the case of supersymmetries and which, to the best of our knowledge, has not been discussed so far in the literature. Namely, it may happen that a symmetry is broken in some pure phase, but there is no corresponding "symmetrized" mixed phase such that an action of generators Q of the symmetry can be defined in it. Thus, whereas the symmetry transformations δ of the fields are still well defined, the idea of transformed state vectors becomes meaningless. We call this case *spontaneous collapse* of symmetry.

In view of the points raised, a thorough discussion of the fate of supersymmetries in thermal states seems desirable. Since it requires a general mathematical setting which may not be so well known, we recall in the first part of the subsequent Sec.2 some relevant mathematical notions and facts, and then turn to the analysis of supercharges. In Sec.3 we discuss the role of superthermal ensembles and of supertraces. The paper concludes with a brief discussion of the physical significance of our results.

2 Status of Supercharges

The assumption that a quantum field theory is supersymmetric implies that there exist Lorentz-covariant anti-local spinorial currents

$$j_{\mu\alpha}(x), \qquad j^{\dagger}_{\nu\dot\beta}(x) \tag{7}$$

which are the hermitian conjugates of each other and are conserved,

$$\partial^{\mu} j_{\mu\alpha}(x) = \partial^{\nu} j^{\dagger}_{\nu\dot\beta}(x) = 0. \tag{8}$$

As is well known, the singular nature of field and current operators at a point, generically denoted by $\varphi(x)$ (with tensor and spinor indices omitted), requires us to smear them with test functions f, i.e., smooth functions on $\mathbb{R}^4$ with compact support,

$$\varphi(f) = \int d^4x \; \varphi(x) f(x).$$

In the following, we use the notation F for polynomials in smeared fields and currents,

$$F = \sum c\, \varphi(f_1)\, \varphi(f_2) \cdots \varphi(f_n), \tag{9}$$

and denote by $\mathcal{F}$ the set of these polynomials (forming an algebra). The space-time translations act on $F \in \mathcal{F}$ by

$$F \mapsto F(x) = \sum c\, \varphi(f_{1,x})\, \varphi(f_{2,x}) \cdots \varphi(f_{n,x}), \quad x \in \mathbb{R}^4 \tag{10}$$

where f_x is obtained from f by setting $f_x(y) = f(y-x),\ y \in \mathbb{R}^4$. Making a choice of Lorentz frame, we write $x = (x_0,\ \boldsymbol{x})$ and use also the shorthand notation for spatial and temporal translates $F(\boldsymbol{x}) = F(0, \boldsymbol{x}), F(x_0) = F(x_0, \mathbf{0})$. We also introduce the notation $\mathcal{F}_{\pm}$ for bosonic/fermionic operators, i.e., polynomials in the smeared field operators containing an even/odd number of fermionic fields in each monomial. Then, we can define the supersymmetry transformations by

$$\delta_\alpha(F_\pm) = \lim_{R\to\infty} \int d^4x \; g(x_0) h(\boldsymbol{x}/R)\; [j_{0\alpha}(x), F_\pm]_\mp \tag{11}$$

and analogously $\bar{\delta}_{\dot{\beta}}$ in terms of $j^{\dagger}_{\nu\dot{\beta}}(x)$, where $F_\pm \in \mathcal{F}_\pm$. Here real test functions $g \in \mathcal{D}(\mathbb{R})$ and $h \in \mathcal{D}(\mathbb{R}^3)$ are so chosen that $\int dx_0\ g(x_0) = 1, h(\boldsymbol{x}) = 1$ for $|\boldsymbol{x}| \leq 1$. From current conservation, Eq.(8), and local (anti-) commutativity, it follows that δ_α and $\bar{\delta}_{\dot{\beta}}$ exist as linear maps acting on $\mathcal{F} = \mathcal{F}_+ + \mathcal{F}_-$ and that they do not depend on the choice of g, h satisfying the stated conditions. Moreover, for given $F_\pm$ the limit in Eq.(11) is attained for some finite R, so the images $\delta_\alpha(F_\pm)$ are again operators belonging to $\mathcal{F}$ in accord with the more formal definition of supersymmetry transformations of fields in the Lagrangian framework. As a matter of fact, it holds that $\delta_\alpha(\mathcal{F}_\pm) \subset \mathcal{F}_\mp$, $\bar{\delta}_{\dot{\beta}}(\mathcal{F}_\pm) \subset \mathcal{F}_\mp$, and hence the mappings can be applied to the elements of $\mathcal{F}$ an arbitrary number of times. They are anti-derivations satisfying the following “graded” Leibniz rule:

$$\delta_\alpha(F_\pm F) = \delta_\alpha(F_\pm)F \pm F_\pm \delta_\alpha(F), \tag{12}$$

for $F_\pm \in \mathcal{F}_\pm, F \in \mathcal{F}$, and similarly for $\bar{\delta}_{\dot{\beta}}$. We also note their behavior under hermitian conjugation following from the hermiticity properties of the currents,

$$\delta_\alpha(F_\pm)^\dagger = \mp\, \bar{\delta}_{\dot{\alpha}}(F_\pm^\dagger). \tag{13}$$

To put the fundamental relation of supersymmetry in a state-independent form, we also introduce the derivation arising from the time translations,

$$\delta_0(F) = -i\frac{d}{dx_0}F(x_0)|_{x_0=0}. \tag{14}$$

Note that the derivation δ_0 and the anti-derivations $\delta_\alpha, \bar{\delta}_{\dot{\beta}}$ commute with the spatial translations, i.e., it holds that $\delta(F(\boldsymbol{x})) = (\delta(F))(\boldsymbol{x})$.

The fundamental relation of supersymmetry can now be expressed as follows:

$$\bar{\delta}_{\dot{1}} \circ \delta_1 + \delta_1 \circ \bar{\delta}_{\dot{1}} + \bar{\delta}_{\dot{2}} \circ \delta_2 + \delta_2 \circ \bar{\delta}_{\dot{2}} = 4\delta_0, \tag{15}$$

where $\circ$ denotes the composition of the respective maps on $\mathcal{F}$. This relation of maps is meaningful independently of the specific choice of a state. It follows either from the very definition of supersymmetry transformations of fields or can be verified in any state

where supercharges can be defined as generators of supersymmetry transformations, for instance, the vacuum. The crucial point is that, because of the fermionic nature of supercharges, cancellations take place in the "mixed terms" where supercharges stand to the left and right of field operators.

We consider now any state $\langle \cdot \rangle$ on $\mathcal{F}$ which is invariant under spatial translations, namely, we assume that $\langle \cdot \rangle$ has the properties $\langle c_1 F_1 + c_2 F_2 \rangle = c_1 \langle F_1 \rangle + c_2 \langle F_2 \rangle$ (linearity), $\langle F^\dagger F \rangle \geq 0$ (positivity), $\langle \mathbf{1} \rangle = 1$ (normalization) and $\langle F(\boldsymbol{x}) \rangle = \langle F \rangle$ (invariance). It is our aim to show that $\langle \cdot \rangle$ must be the vacuum if supersymmetry is not broken in this state. The argument will be given in several steps.

(a) By the reconstruction theorem [7], there exists a Hilbert space $\mathfrak{H}$ and a distinguished unit vector $|1\rangle$ such that the set of vectors $F\,|1\rangle$, $F \in \mathcal{F}$, is dense in $\mathfrak{H}$ and

$$\langle F \rangle = \langle 1 |\, F \,| 1 \rangle \tag{16}$$

holds for any $F \in \mathcal{F}$.[3] First, we show that the Bose-Fermi superselection rule is not broken spontaneously in any such state. Let $F_- \in \mathcal{F}_-$ be a fermionic operator. From our assumption of the invariance of $\langle \cdot \rangle$ under spatial translations it follows that $\langle F_- \rangle = \langle F_-(\boldsymbol{x}) \rangle = \langle 1 |\, F_-(\boldsymbol{x}) \,| 1 \rangle = (1/|V|) \int_V d^3\boldsymbol{x}\ \langle 1 |\, F_-(\boldsymbol{x}) \,| 1 \rangle$, where V denotes any bounded spatial region in $\mathbb{R}^3$ and $|V|$ its volume. In the limit of $V \nearrow \mathbb{R}^3$ the right-hand side of this equality can be shown to vanish because of the following bound on the norm of the spatial mean of fermionic vectors,

$$\begin{aligned}
& \| \frac{1}{|V|} \int_V d^3\boldsymbol{x} F_-(\boldsymbol{x})\,|1\rangle \|^2 + \| \frac{1}{|V|} \int_V d^3\boldsymbol{x} F_-(\boldsymbol{x})^\dagger\,|1\rangle \|^2 \\
= \; & \frac{1}{|V|} \int_V d^3\boldsymbol{x}\, \frac{1}{|V|} \int_V d^3\boldsymbol{y}\ \langle\, [\, F_-(\boldsymbol{x})^\dagger,\, F_-(\boldsymbol{y})\,]_+ \,\rangle \\
\leq \; & \frac{1}{|V|} \int d^3\boldsymbol{z}\ |\, \langle\, [\, F_-(\boldsymbol{z})^\dagger, F_-\,]_+ \,\rangle\, |,
\end{aligned} \tag{17}$$

where we made use of the invariance of $\langle \cdot \rangle$ under spatial translations. Note that the latter integral exists since the anti-commutator vanishes for large spatial translations $\boldsymbol{z}$.

[3] Strictly speaking, one should distinguish between the "abstract" elements $F \in \mathcal{F}$ and their concrete realization as operators on $\mathfrak{H}$ which depends on the given state $\langle \cdot \rangle$. Since there is no danger of confusion, we use the present simplified notation.

From this, we conclude that

$$\langle F_- \rangle = 0 \quad \text{for } F_- \in \mathcal{F}_- \tag{18}$$

which asserts the validity of the Bose-Fermi superselection rule.

(b) We say that supersymmetry is *implementable in the state* $\langle \cdot \rangle$ if there exist operators Q_α and $Q^\dagger_{\dot\beta}$ (hermitian conjugate of Q_β) which have the vectors $F\,|1\rangle, F \in \mathcal{F}$, in their domains of definition and satisfy

$$Q_\alpha F_\pm\,|1\rangle \;=\; \delta_\alpha(F_\pm)\,|1\rangle \pm F_\pm Q_\alpha\,|1\rangle, \tag{19}$$

$$Q^\dagger_{\dot\beta} F_\pm\,|1\rangle \;=\; \bar\delta_{\dot\beta}(F_\pm)\,|1\rangle \pm F_\pm Q^\dagger_{\dot\beta}\,|1\rangle. \tag{20}$$

Remarks: (i) We do not assume from the outset that $Q_\alpha\,|1\rangle = Q^\dagger_{\dot\beta}\,|1\rangle = 0$ or that Q_α and $Q^\dagger_{\dot\beta}$ commute with translations because of the ambiguities involved in the definition of generators in the case of thermal states, cf. Sec.1.

(ii) Picking arbitrary vectors $|\alpha\rangle, |\dot\beta\rangle$ in the domain of all operators in $\mathcal{F}$, one can always (i.e., irrespective of the occurrence of spontaneous symmetry breakdown) define consistently linear operators $\hat Q_\alpha, \check Q_{\dot\beta}$ if $|1\rangle$ has the property of being separating for $\mathcal{F}$, i.e., if $F\,|1\rangle = 0$ implies $F = 0$ for $F \in \mathcal{F}$. (This property holds for vacuum states by the Reeh-Schlieder theorem [7] and also for thermal equilibrium states as a consequence of the KMS condition [5], cf. below). One simply puts

$$\hat Q_\alpha F_\pm\,|1\rangle \;=\; \delta_\alpha(F_\pm)\,|1\rangle \pm F_\pm|\alpha\rangle, \tag{21}$$

$$\check Q_{\dot\beta} F_\pm\,|1\rangle \;=\; \bar\delta_{\dot\beta}(F_\pm)\,|1\rangle \pm F_\pm\;|\dot\beta\rangle. \tag{22}$$

In this formulation, spontaneous breakdown of supersymmetries means that, for no choice of $|\alpha\rangle, |\dot\alpha\rangle$, the operators $\hat Q_\alpha, \check Q_{\dot\alpha}$ are the hermitian conjugates of each other. They then have very pathological properties (e.g., are not closable [9]) and thus are not physically acceptable.

We will now show that, if supersymmetry is implementable in the state $\langle \,\cdot\, \rangle$ in the sense specified above, then it holds that

$$\langle \delta_\alpha(\,\cdot\,) \rangle = \langle \bar{\delta}_{\dot\beta}(\,\cdot\,) \rangle = 0, \tag{23}$$

i.e., the state $\langle \,\cdot\, \rangle$ passes the familiar test for symmetry. It follows from the Bose-Fermi superselection rule that, for any $F_+ \in \mathcal{F}_+$,

$$\langle \delta_\alpha(F_+) \rangle = \langle \bar{\delta}_{\dot\beta}(F_+) \rangle = 0, \tag{24}$$

because of $\delta_\alpha(F_+), \bar{\delta}_{\dot\beta}(F_+) \in \mathcal{F}_-$. In order to show that these expressions vanish also for fermionic operators $F_- \in \mathcal{F}_-$,

$$\langle \delta_\alpha(F_-) \rangle = \langle \bar{\delta}_{\dot\beta}(F_-) \rangle = 0, \tag{25}$$

we make use of Eqs.(19), (20). Combining these relations and the commutativity between the anti-derivation δ_α and spatial translations $\boldsymbol{x}$, we obtain, as in step (a),

$$\begin{aligned} \langle \delta_\alpha(F_-) \rangle &= \frac{1}{|V|} \int_V d^3\boldsymbol{x} \; \langle 1|\, \delta_\alpha(F_-(\boldsymbol{x}))\, |1\rangle \\ &= \frac{1}{|V|} \int_V d^3\boldsymbol{x} \; \langle 1|\, (Q_\alpha F_-(\boldsymbol{x}) + F_-(\boldsymbol{x}) Q_\alpha)\, |1\rangle. \end{aligned} \tag{26}$$

Hence, by making use of the Cauchy-Schwarz inequality, we arrive at the estimate

$$|\langle \delta_\alpha(F_-) \rangle| \leq \| Q_\alpha^\dagger \,|1\rangle \| \cdot \| \frac{1}{|V|} \int_V d^3x \; F_-(\boldsymbol{x})\, |1\rangle \| + \| \frac{1}{|V|} \int_V d^3x \; F_-(\boldsymbol{x})^\dagger \,|1\rangle \| \cdot \| Q_\alpha \,|1\rangle \| \tag{27}$$

for arbitrary V. The right-hand side of this inequality vanishes for $V \nearrow \mathbb{R}^3$ according to relation (17) which shows that $\langle \delta_\alpha(F_-) \rangle = 0$ for $F_- \in \mathcal{F}_-$. By the same token we obtain also $\langle \bar{\delta}_{\dot\beta}(F_-) \rangle = 0$, $F_- \in \mathcal{F}_-$, which completes the proof of relation (25). As a consequence of the fundamental relation Eq.(15) characterizing supersymmetry, the invariance of $\langle \,\cdot\, \rangle$ under time translations automatically follows:

$$\langle \delta_0(\,\cdot\,) \rangle = 0. \tag{28}$$

We emphasize that, in the above discussion, we did not assume the cluster property with respect to spatial translations, and hence, the result is valid even if the state $\langle \,\cdot\, \rangle$ describes an arbitrary mixed thermodynamic phase. The only condition on $\langle \,\cdot\, \rangle$ is that supercharges can be defined.

(c) In the next step, we show that, if a state $\langle \,\cdot\, \rangle$ is supersymmetric in the sense of equation (23) and complies with the Bose-Fermi superselection rule (18), then it is a vacuum state. More precisely, it is invariant under space and time translations and there are corresponding generators satisfying the relativistic spectrum condition (positivity of energy in all Lorentz frames). To prove this statement, we consider the expectation values $\langle F^\dagger \delta_0(F)\rangle$ for $F \in \mathcal{F}$. Since $F = F_+ + F_-$ with $F_\pm \in \mathcal{F}_\pm$ and $\langle F_\pm^\dagger \delta_0(F_\mp)\rangle = 0$ by (18), we may restrict our attention to the expectation values $\langle F_\pm^\dagger \delta_0(F_\pm)\rangle$. According to the fundamental relation of supersymmetry, it holds that

$$4\langle F_+^\dagger \, \delta_0(F_+)\rangle = \langle F_+^\dagger \, (\bar{\delta}_{\dot 1} \circ \delta_1 + \delta_1 \circ \bar{\delta}_{\dot 1} + \bar{\delta}_{\dot 2} \circ \delta_2 + \delta_2 \circ \bar{\delta}_{\dot 2})(F_+)\rangle, \tag{29}$$

and we take a close look at the terms appearing on the right-hand side. Making use of the fact that the $\delta_\alpha, \bar{\delta}_{\dot\beta}$ are anti-derivations and of Eq.(13), we have

$$\begin{aligned} \bar{\delta}_{\dot 1}(F_+^\dagger \, \delta_1(F_+)) &= \bar{\delta}_{\dot 1}(F_+^\dagger) \, \delta_1(F_+) + F_+^\dagger \, \bar{\delta}_{\dot 1}(\delta_1(F_+)) \\ &= -\delta_1(F_+)^\dagger \, \delta_1(F_+) + F_+^\dagger \, \bar{\delta}_{\dot 1} \circ \delta_1(F_+). \end{aligned} \tag{30}$$

Since $\langle \, \bar{\delta}_{\dot 1}(\,\cdot\,) \, \rangle = 0$, therefore, we find that

$$\langle F_+^\dagger \, \bar{\delta}_{\dot 1} \circ \delta_1(F_+)\rangle = \langle \delta_1(F_+)^\dagger \, \delta_1(F_+)\rangle \geq 0 \tag{31}$$

and a similar argument applies to the remaining terms. So $\langle F_+^\dagger \, \delta_0(F_+)\rangle \geq 0$ and the same result holds if one replaces F_+ by $F_- \in \mathcal{F}_-$. Putting together all this, we arrive at

$$\langle F^\dagger \, \delta_0(F)\rangle \geq 0 \qquad \text{for } F \in \mathcal{F}. \tag{32}$$

Now, since δ_0 is a derivation satisfying the Leibniz rule (3), $\delta_0(F)^\dagger = -\delta_0(F^\dagger)$ for $F \in \mathcal{F}$, and $\langle \delta_0(\,\cdot\,)\rangle = 0$, the operator P_0 given by

$$P_0 F\,|1\rangle = \delta_0(F)\,|1\rangle \qquad \text{for } F \in \mathcal{F} \tag{33}$$

is well defined and hermitian[4] and satisfies $P_0\,|1\rangle = \delta_0(\mathbf{1})|1\rangle = 0$. From the lower bound

$$\langle 1|\, F^\dagger\, P_0\, F\,|1\rangle = \langle F^\dagger \delta_0(F)\rangle \geq 0, \tag{34}$$

it follows that P_0 is a positive operator, so, in view of $P_0\,|1\rangle = 0$, we conclude that $|1\rangle$ is a ground state for P_0.

To show that the state $\langle\,\cdot\,\rangle$ is a ground state in any Lorentz frame, we make use of the spinorial transformation properties of the supercurrents. They imply that the fundamental relation (15) holds for the transformed maps ${\delta_\alpha}', {\bar{\delta}_{\dot\beta}}'$ and ${\delta_0}'$ in any Lorentz frame. Moreover, since a change of Lorentz frame amounts to a linear transformation of these maps, i.e.,

$${\delta_\alpha}' = {A_\alpha}^\beta \delta_\beta, \quad {\bar{\delta}_{\dot\alpha}}' = \overline{{A_\alpha}^\beta}\,\bar{\delta}_{\dot\beta} = {\bar{A}_{\dot\alpha}}{}^{\dot\beta}\bar{\delta}_{\dot\beta} \tag{35}$$

with $A \in SL(2,\mathbb{C})$, it follows that, if supersymmetry is unbroken in some Lorentz frame in the sense of Eq.(23), this holds true in any other frame. Applying the preceding arguments to the primed transformations, we arrive at the conclusion that the corresponding generators ${P_0}'$ of time translations are positive in all Lorentz frames and satisfy ${P_0}'\,|1\rangle = 0$. Hence $\langle\,\cdot\,\rangle$ is a vacuum state, as claimed.

(d) Let us finally demonstrate that, in spite of possible ambiguities involved in the definition of generators and the ensuing interpretation of $\langle\,\cdot\,\rangle$, this state definitely does not describe a thermal equilibrium situation. To this end we show that $\langle\,\cdot\,\rangle$ does not satisfy the KMS condition for any finite temperature $\beta^{-1} > 0$. Although the argument is standard, we present it here for the sake of completeness. Let P_0 be the non-negative

[4]Making use of temperedness of the underlying fields, one can show that P_0 is even essentially self-adjoint on its domain of definition.

generator defined in the preceding step. Then, there holds for all operators of the form $F(g) = \int dx_0 \; g(x_0) F(x_0)$ the equality

$$F(g)\,|1\rangle = (2\pi)^{1/2} \tilde{g}(P_0) F\,|1\rangle \tag{36}$$

where $\tilde{g}$ is the Fourier transform of g. Hence $F(g)\,|1\rangle = 0$ if $\tilde{g}$ has its support on the negative real axis. Now, if $\langle \,\cdot\, \rangle$ satisfies the KMS condition for some β, it follows that for $F_1, F_2 \in \mathcal{F}$ we can continue analytically the function $x_0 \mapsto \langle\, F_1\,(F_2^\dagger\, F(g))(x_0)\,\rangle$ to a function analytic in the complex domain $\{z \in \mathbb{C} \,:\, 0 < \mathrm{Im} z < \beta\}$ whose boundary value at $\mathrm{Im} z = \beta$ is given by $x_0 \mapsto \langle\, (F_2^\dagger\, F(g))(x_0)\, F_1\,\rangle$. Since there holds $\langle\, F_1\,(F_2^\dagger\, F(g))(x_0)\,\rangle = \langle 1|\, F_1\, F_2^\dagger(x_0)\, F(g)(x_0)\,|1\rangle = 0$ for all $x_0 \in \mathbb{R}$, we find that $\langle\, F_2^\dagger\, F(g)\, F_1\,\rangle = \langle 1|\, F_2^\dagger\, F(g)\, F_1\,|1\rangle = 0$. As F_1, F_2 are arbitrary, we are thus led to the conclusion that $F(g) = 0$. By applying the same argument to $F^\dagger(g)$, we obtain similarly $F^\dagger(g) = 0$ and hence $F(\bar{g}) = F^\dagger(g)^\dagger = 0$. Hence, $F(g) = 0$ for any g whose Fourier transform vanishes in some neighborhood of the origin. Therefore, the operator function $x_0 \mapsto F(x_0)$ is a polynomial in x_0 which can only be constant because of $\langle\, F(x_0)^\dagger F(x_0)\,\rangle = \langle\, F^\dagger F\,\rangle$ by time invariance of $\langle \,\cdot\, \rangle$. Thus, since we can exclude the case of trivial dynamics, i.e., $F(x_0) = F$ for all $F \in \mathcal{F}$ and $x_0 \in \mathbb{R}$, the assumption that $\langle \,\cdot\, \rangle$ satisfies the KMS condition for some β leads to a contradiction.

Let us summarize: The existence of generators of supersymmetry (supercharges) in an arbitrary spatially homogeneous state $\langle \,\cdot\, \rangle$ implies that this state is supersymmetric in the sense of relation (23) and that the Bose-Fermi superselection rule is unbroken in this state. But any state with these two properties is necessarily a vacuum state and does not satisfy the KMS condition for finite β. Thus supersymmetry is broken in all thermal equilibrium states, irrespective of whether they describe pure or mixed homogeneous phases. Moreover, generators of supersymmetry cannot be defined in such states. Hence, thermal effects induce an inevitable spontaneous collapse of supersymmetry.

3 Role of Supertrace

In discussions of thermal properties of supersymmetric theories, one frequently encounters so-called superthermal ensembles, described by non-positive "density matrices". It has been pointed out by van Hove [10] that thermal averages in these ensembles, called supertraces $s(\,\cdot\,)$ in the following, ought to be interpreted as weighted differences of the underlying bosonic and fermionic subensembles,

$$s(\,\cdot\,) = w_b \;\langle\,\cdot\,\rangle_b - w_f \;\langle\,\cdot\,\rangle_f. \tag{37}$$

Here $\langle\,\cdot\,\rangle_b$, $\langle\,\cdot\,\rangle_f$ are the corresponding physical states and w_b, w_f are non-negative numbers. Whenever this decomposition is meaningful, one can normalize these numbers according to $w_b + w_f = 1$.

It is sometimes argued [3] that the behavior of $s(\,\cdot\,)$ under the action of the supersymmetry transformations δ_α, $\bar{\delta}_{\dot\beta}$ provides the appropriate test for the spontaneous breakdown of supersymmetries. If $s(\delta_\alpha(\,\cdot\,)) = s(\bar{\delta}_{\dot\beta}(\,\cdot\,)) = 0$, supersymmetry is said to be unbroken, otherwise it is regarded as spontaneously broken. This interpretation has no convincing conceptual basis, however, and it is therefore of some interest to explore the actual physical meaning of the two cases. Again the issue becomes very clear in the thermodynamic limit. It turns out that there exist only the following two possibilities in a spatially homogeneous situation: Either $s(\,\cdot\,) = 0$ (i.e., the supertrace is trivial) or the functionals $s(\delta_\alpha(\,\cdot\,))$ and $s(\bar{\delta}_{\dot\beta}(\,\cdot\,))$ are different from 0. Thus, superthermal ensembles can be supersymmtric only if they are trivial (having a zero "density matrix").

Before going into the proof of this statement, let us briefly discuss its physical meaning. If $s(\,\cdot\,) = 0$, then $w_b = w_f$ and $\langle\,\cdot\,\rangle_b = \langle\,\cdot\,\rangle_f$, i.e., one cannot distinguish between a "bosonic" and a "fermionic" phase. We emphasize that this does not imply the existence of only a single phase, because the state $\langle\,\cdot\,\rangle_b = \langle\,\cdot\,\rangle_f$ may well describe a mixture of different phases. On the other hand, if $s(\,\cdot\,) \neq 0$, there are two possibilities; either (i) there are at least two different phases or (ii) there holds $\langle\,\cdot\,\rangle_b = \langle\,\cdot\,\rangle_f$ and

$w_b \neq w_f$. It has been argued in [10] that the latter case does not occur in situations of physical interest. Yet since this argument is based on the existence of supercharges it is not applicable in the thermodynamic limit. In order to establish in this way the existence of different phases, therefore, one has to carry out further tests on $s(\,\cdot\,)$. (It would be sufficient, for instance, to show that the superaverages of positive operators, $s(F^\dagger F)$, can attain positive and negative values for suitable choices of $F \in \mathcal{F}$.)

Thus, the supertrace may be used to obtain information about the phase structure of supersymmetric theories. Apart from the trivial case $s(\,\cdot\,) = 0$, however, there is no restoration of supersymmetry at finite temperature even in the sense of the supertrace.

We prove the above statement on $s(\,\cdot\,)$ in two steps. First, we assume that both the bosonic and fermionic subensembles are pure phases. Then, as was explained in Sec.1, they have the cluster property, which we use here in a somewhat weaker form

$$\frac{1}{|V|}\int_V d^3\boldsymbol{x}\ \langle\, F_1(\boldsymbol{x})F_2\,\rangle_{b,f} - \langle\, F_1\,\rangle_{b,f}\langle\, F_2\,\rangle_{b,f} \quad \rightarrow \quad 0 \qquad \text{as } V \nearrow \mathbb{R}^3. \tag{38}$$

If $s(\,\cdot\,)$ is invariant under supersymmetry transformations, $s(\delta_\alpha(\,\cdot\,)) = 0$, we obtain from the graded Leibniz rule (12)

$$0 = s(\delta_\alpha(F_-(\boldsymbol{x})\,F_+)) = s(\delta_\alpha(F_-(\boldsymbol{x}))\,F_+ - F_-(\boldsymbol{x})\,\delta_\alpha(F_+)) \tag{39}$$

for any $F_\pm \in \mathcal{F}_\pm$. Because of the decomposition (37) of $s(\,\cdot\,)$, this equality can be rewritten in the form

$$w_b\ \langle\delta_\alpha(F_-(\boldsymbol{x}))\,F_+ - F_-(\boldsymbol{x})\,\delta_\alpha(F_+)\rangle_b = w_f\ \langle\delta_\alpha(F_-(\boldsymbol{x}))\,F_+ - F_-(\boldsymbol{x})\,\delta_\alpha(F_+)\rangle_f. \tag{40}$$

Bearing in mind that $\delta_\alpha(\,\cdot\,)$ commutes with spatial translations, we thus obtain, by taking a spatial mean on both sides of (40) and making use of the cluster property (38),

$$w_b\ \Big(\langle\delta_\alpha(F_-)\rangle_b\,\langle F_+\rangle_b - \langle F_-\rangle_b\,\langle\delta_\alpha(F_+)\rangle_b\Big) = w_f\ \Big(\langle\delta_\alpha(F_-)\rangle_f\,\langle F_+\rangle_f - \langle F_-\rangle_f\,\langle\delta_\alpha(F_+)\rangle_f\Big). \tag{41}$$

From the Bose-Fermi superselection rule (18) applied to $\langle \cdot \rangle_{b,f}$, we get $\langle F_- \rangle_{b,f} = 0$ and hence (41) reduces to

$$w_b \, \langle \delta_\alpha(F_-) \rangle_b \, \langle F_+ \rangle_b = w_f \, \langle \delta_\alpha(F_-) \rangle_f \, \langle F_+ \rangle_f. \tag{42}$$

The normalization condition $\langle \mathbf{1} \rangle_b = \langle \mathbf{1} \rangle_f = 1$ implies $w_b \, \langle \delta_\alpha(F_-) \rangle_b = w_f \, \langle \delta_\alpha(F_-) \rangle_f$; moreover, since $\langle \cdot \rangle_{b,\,f}$ are thermal states, there exist, by the results obtained in Sec.2, operators $F_- \in \mathcal{F}_-$ for which $\langle \delta_\alpha(F_-) \rangle_{b,\,f} \neq 0$. Therefore, relation (42) implies that $\langle \cdot \rangle_b = \langle \cdot \rangle_f$ and $w_b = w_f$ and hence we arrive at $s(\,\cdot\,) = 0$.

Next, we discuss the general case where $\langle \cdot \rangle_{b,\,f}$ describe mixtures of thermodynamic phases. Then, we decompose the state $\langle \cdot \rangle = w_b \, \langle \cdot \rangle_b + w_f \, \langle \cdot \rangle_f$ into pure phases $\langle \cdot \rangle_\theta$ (central decomposition) as described in Sec.1, and get in particular

$$w_b \, \langle \cdot \rangle_b \;=\; \sum_\theta w_b(\theta) \, \langle \cdot \rangle_\theta, \tag{43}$$

$$w_f \, \langle \cdot \rangle_f \;=\; \sum_\theta w_f(\theta) \, \langle \cdot \rangle_\theta, \tag{44}$$

where $w_b(\theta)$ and $w_f(\theta)$ are, respectively, non-negative weight factors and $\sum_\theta w_b(\theta) = w_b$, $\sum_\theta w_f(\theta) = w_f$.

As in the case of pure phases, we proceed from the assumption of $s(\delta_\alpha(\,\cdot\,)) = 0$ to relation (40), where we now insert the decompositions (43), (44) of the bosonic and fermionic subensembles. Taking a spatial mean of the resulting expression and proceeding to the limit $V \nearrow \mathbb{R}^3$, we obtain, by applying the cluster property to each component pure phase $\langle \cdot \rangle_\theta$, the relation

$$\sum_\theta w_b(\theta) \, \langle \delta_\alpha(F_-) \rangle_\theta \, \langle F_+ \rangle_\theta = \sum_\theta w_f(\theta) \, \langle \delta_\alpha(F_-) \rangle_\theta \, \langle F_+ \rangle_\theta. \tag{45}$$

To be precise, the interchange of the limit $V \nearrow \mathbb{R}^3$ with the summation $\sum_\theta$ requires some justification in the cases of an infinite number of phases, or of a continuum of phases (where, instead of the summation, an integration appears with a suitable probability measure). We refrain from presenting these technical details here.

If one replaces in (45) the operator F_+ by $(1/|V|)\int_V d^3\boldsymbol{x}\ (\delta_\alpha(F_-)^\dagger)(\boldsymbol{x})\, F_+$ and makes use again of the cluster properties of pure phases, one obtains in the limit $V \nearrow \mathbb{R}^3$ the relation

$$\sum_\theta w_b(\theta)\ |\langle\, \delta_\alpha(F_-)\,\rangle_\theta|^2\,\langle\, F_+\,\rangle_\theta = \sum_\theta w_f(\theta)\ |\langle\, \delta_\alpha(F_-)\,\rangle_\theta|^2\,\langle\, F_+\,\rangle_\theta. \tag{46}$$

Repeating this procedure, one arrives at similar relations involving higher products of expectation values of arbitrary operators F_+ in the pure phases $\langle\ \cdot\ \rangle_\theta$. The resulting constraints on the weight factors $w_b(\theta), w_f(\theta)$ become obvious if one interprets (46) as a relation between (non-normalized) states. Keeping F_- fixed and varying F_+, one sees that the two functionals $\sum_\theta w_{b,\,f}(\theta)\ |\langle\, \delta_\alpha(F_-)\,\rangle_\theta|^2\,\langle\ \cdot\ \rangle_\theta$ coincide. Because of the uniqueness of the central decomposition this implies for any θ the equality

$$w_b(\theta)\ |\langle\, \delta_\alpha(F_-)\,\rangle_\theta|^2 = w_f(\theta)\ |\langle\, \delta_\alpha(F_-)\,\rangle_\theta|^2. \tag{47}$$

Since $\langle\ \cdot\ \rangle_\theta$ is a thermal state, however, there is some $F_- \in \mathcal{F}_-$ such that $\langle\, \delta_\alpha(F_-)\,\rangle_\theta \neq 0$ and hence we obtain $w_b(\theta) = w_f(\theta)$. It then follows from relations (43), (44) that $s(\ \cdot\ ) = 0$, so only the trivial supertrace is supersymmetric.

Thus we conclude that the supertrace is a device to deduce some (partial) information about the phase structure in supersymmetric theories. Yet its behaviour under supersymmetry transformations does not provide any additional information, in accord with the result of the previous section that supersymmetry always suffers from a spontaneous collapse in thermal states.

4 Conclusions

In the present article, we have clarified in a general setting the status of supersymmetry in thermal states. In every quantum field theory where an action of supersymmetry transformations on the fields can be so defined that the fundamental relation (15) holds, this symmetry suffers from a spontaneous collapse in thermal states. We have

established this result for spatially homogeneous states in d=4 dimensions; but it can easily be extended to more complex situations (such as asymptotically homogeneous states, spatially periodic states, etc.) and to any number of spacetime dimensions. Moreover, the point-like nature and strict local (anti-) commutativity of the underlying fields is not really crucial. Our arguments require only a sufficiently rapid fall-off of the expectation values of (anti-) commutators of the underlying field operators for large spatial translations. Therefore, an analogous result may be expected to hold in quantum superstring field theory, provided a pertinent formulation of supersymmetry can be given in that setting.

The universal breakdown of certain symmetries in thermal states is a well known phenomenon. A prominent example is the Lorentz symmetry which is inevitably broken in thermal equilibrium states, since the KMS condition fixes a rest frame [11]. Nevertheless, an action of Lorentz transformations can be defined on thermal states and is physically meaningful: A gas which is macroscopically at rest in a given Lorentz frame is transformed into a gas in motion with respect to that frame, etc. (cf. [12] for a general characterization of thermal equilibrium states in arbitrary Lorentz frames).

This familiar situation of spontaneous breakdown of a symmetry should be clearly distinguished from the spontaneous collapse of supersymmetry in thermal states, where it is no longer possible to define an action of the symmetry on the physical states.

In view of this vulnerability to thermal effects, one may wonder how supersymmetry can manifest itself in real physical systems. The theoretical prediction of a zero energy mode in thermal states is of limited value, since this mode need not be affiliated with a Goldstino particle, but may result from long range correlations between particle-hole pairs [13]. Also, rigorous results on the fate of particle supermultiplets in a thermal environment do not exist yet.

For a reliable prediction of the existence of supersymmetry in physical systems, it seems necessary to show that symmetry properties of the vacuum theory can be

recovered from thermal states in the limit of zero temperature. On the other hand, the possibility that supersymmetry remains collapsed in this limit, in analogy to some hysteresis effect, may be even more interesting since it could account for the apparent absence of this symmetry in the real world. It would therefore be desirable to clarify which of these two possibilities is at hand in models of physical interest.

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References

[1] L. Girardello, M. Grisaru and P. Salomonson, Nucl. Phys. **B178** (1981) 331;
D. Boyanovsky, Phys. Rev. **D29** (1984) 743.

[2] J. Fuchs, Nucl. Phys. **B246** (1984) 279;
Won-Ho Kye, Sin Kyu Kang and Jae Kwan Kan, Phys. Rev. **D46** (1992) 1835

[3] L. van Hove, Nucl. Phys. **B207** (1982) 15;
T.E. Clark and S.T. Love, Nucl. Phys. **B217** (1983) 349.

[4] R. Haag, N.M. Hugenholtz and M. Winnink, Comm. Math. Phys. **5** (1967) 215;
I. Ojima, Ann. Phys. **137** (1981) 1.

[5] O. Bratteli and D.W. Robinson, Operator Algebras and Quantum Statistical Mechanics, Vols. 1 and 2 (Springer, Berlin, 1979 and 1981).

[6] H. Matsumoto, M. Tachiki and H. Umezawa, Thermo Field Dynamics and Condensed States (North-Holland, Amsterdam, 1982).

[7] R.F. Streater and A.S. Wightman, PCT, Spin and Statistics and All That (Benjamin, New York, 1964);
R. Haag, Local Quantum Physics (Springer, Berlin, 1992[1st ed.], 1996[2nd ed.]).

[8] M.B. Paranjape, A. Taormina and L.C.R. Wijewardhana, Phys. Rev. Lett. **50** (1983) 1350.

[9] H. Reeh, Lecture Notes in Physics **39** (H. Araki, ed.) (Springer, Berlin, 1975) pp.249-255.

[10] L. van Hove, Fizika **17** (1985) 267.

[11] I. Ojima, Lecture Notes in Physics **176** (K. Kikkawa, et al. eds.) (Springer, Berlin, 1983) pp.161-165; Lett. Math. Phys. **11** (1986) 73.

[12] J. Bros and D. Buchholz, Nucl. Phys. **B429** (1994) 291.

[13] H. Matsumoto, N.J. Papastamatiou, H. Umezawa and N. Yamamoto, Phys. Rev. **D34** (1986) 3217.